

**MODELING AND CONTROLLER DESIGN FOR AN INVERTED  
PENDULUM SYSTEM**

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To my dearest father, mother and family,  
for their encouragement, blessing and inspiration ...

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## **ABSTRACT**

This paper presents the simulation study of several control strategies for an inverted pendulum system. The goal is to determine which control strategy delivers better performance with respect to pendulum's angle and cart's position. The inverted pendulum represents a challenging control problem, which continually moves toward an uncontrolled state. Three controllers are presented i.e. proportional-integral-derivative (PID), linear quadratic regulator (LQR) for controlling the linear system of inverted pendulum and fuzzy logic controller (FLC) for controlling the non-linear system of inverted pendulum model. Simulation study has been done in Simulink shows that LQR produced better response compared to PID and FLC control strategies and offers considerable robustness.

## **ABSTRAK**

Dalam thesis ini, beberapa teknik kawalan untuk pendulum songsang telah dikaji. Tujuannya adalah untuk menentukan teknik kawalan yang mempunyai prestasi terbaik terhadap sudut pendulum dan kedudukan kereta. Pendulum songsang adalah satu masalah dalam bidang kawalan yang mencabar yang mana secara berterusan menuju ke arah keadaan yang tidak terkawal. Tiga pengawal dibincangkan iaitu PID, LQR untuk mengawal sistem pendulum songsang yang liner dan FLC untuk mengawal sistem pendulum songsang yang kompleks. Simulasi telah dibuat dengan menggunakan perisian Simulink dimana pengawal LQR menghasilkan sambutan masa terbaik berbanding dengan teknik pengawalan PID dan teknik pengawalan FLC. Teknik pengawalan LQR mempunyai kestabilan yang memuaskan.

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## LIST OF SYMBOLS

SYMBOL		DESCRIPTION
$b$	-	Friction or cart
$l$	-	Length to pendulum centre of mass
$x$	-	Cart position coordinate
$\ddot{x}$	-	cart acceleration
$\theta$	-	Pendulum angle from the vertical
$\ddot{\theta}$	-	Pendulum angular acceleration
$r(s)$	-	Reference signal
$e(s)$	-	Error signal
$u(s)$	-	Plant input
$g(s)$	-	Plant
$y(s)$	-	Output, pendulum's angle
$f(s)$	-	Disturbance, force
$x$	-	State vector
$u$	-	Input vector
$y$	-	Output vector
$t_s$	-	Settling time
$t_r$	-	Rising time
$e_{ss}$	-	Steady state error
$K_p$	-	Error multiplied by a gain
$K_i$	-	The integral of error multiplied by a gain
$K_d$	-	The rate of change of error multiplied by a gain

$K_D(s)$	-	Controller gain
$X(s)$	-	Cart's position signal
NS	-	Negative small
NM	-	Negative medium
NL	-	Negative large
ZE	-	Zero
PS	-	Positive small
PM	-	Positive medium
PL	-	Positive Large
PID	-	Proportional Integral Derivative
LQR	-	Linear Quadratic Regulator
FLC	-	Fuzzy Logic Controller
ANN	-	Artificial Neural Network Controller
AI	-	Artificial Intelligence
GUI	-	Graphic User Interface
M	-	Mass of cart
M	-	Mass of pendulum
CV	-	Control variable
E	-	Error
SP	-	Set point
PV	-	Process variable
SISO	-	Single-input-single-output
R	-	Step input to the cart
A	-	State matrix
B	-	Input matrix
C	-	Output matrix
D	-	Direct transmission matrix
%OS	-	Percent overshoot
I	-	Inertia of the pendulum
F	-	Force applied to cart



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## **CHAPTER 1**

### **INTRODUCTION**

#### **1.1 Overview**

The inverted pendulum offers a very good example for control engineers to verify a modern control theory. This can be explained by the facts that inverted pendulum is marginally stable, in control sense, has distinctive time variant mathematical model.

Inverted Pendulum is a very good model for the attitude control of a space booster rocket and a satellite, an automatic aircraft landing system, aircraft stabilization in the turbulent air-flow, stabilization of a cabin in a ship etc. To solve such problem with non-linear time variant system, there are alternatives such as real time computer simulation of these equations or linearization.

The inverted pendulum is a highly nonlinear and open-loop unstable system. This means that standard linear techniques cannot model the nonlinear dynamics of the system. When the system is simulated the pendulum falls over quickly. The

characteristics of the inverted pendulum make identification and control more challenging.

The inverted pendulum is an intriguing subject from the control point of view due to their intrinsic nonlinearity. The problem is to balance a pole on a mobile platform that can move in only two directions, to the left or to the right. This control problem is fundamentally the same as those involved in rocket or missile propulsion. Common control approaches such as Proportional-Integral-Derivative (PID) control and Linear Quadratic control (LQ) require a good knowledge of the system and accurate tuning in order to obtain desired performances. However, it is often impossible to specify an accurate mathematical model of the process, or the description with differential equations is extremely complex. [3]

In order to obtain control surface, the inverted pendulum dynamics should be locally linearized. Moreover, application of these control techniques to a two or three stage inverted pendulum may result in a very critical design of control parameters and difficult stabilization. However, using artificial intelligence controllers such as artificial neural network and fuzzy logic controllers, the controller can be design without require the model to be linearized. The non-linearized model can be simulated directly using the Matlab application to see result. Therefore, in this project, four types of controllers will be simulated. These four controllers can be divided into two categories.

(1) Conventional Controller

Proportional Integral Derivative (PID)

Linear Quadratic Regulator (LQR)

(2) Artificial Intelligence Controller

Fuzzy Logic Controller (FLC)

Artificial Neural Network Controller (ANN)

### 1.1.1 Why Choose the Inverted Pendulum

The following reasons help explain why the inverted pendulum on a cart has been selected as the system on which the findings of this report will be implemented.

1. A progressive model can be built. It is a non-linear system, yet can be approximated as a linear system if the operating range is small (i.e. slight variations of the angle from the norm).
2. Intuition plays a large part in the human understanding of the inverted pendulum model. When the control method is supplemented with a fuzzy logic and artificial neural network optimization techniques, the result will provide an insightful measure of the ability of the method to provide control.
3. The cart/pole system is a common test case for fuzzy logic so any results can be compared to previous work in the field. In order to perform sound criticism of any controllers developed, a reference model must be designed at the outset of this work. If any testing is worth doing at all it must be planned in such a way that it has at least a good chance of giving a useful result [2]. A proportional, integral, derivative (PID) and LQR controllers will be used as a reference because both the structure of the controller is simple and the performance is not adversely affected by noise and parameter variations.

## 1.2 Objectives

This project consists of three objectives as listed below:

- i. To design artificial intelligence (AI) controller (Fuzzy Logic controller, FLC and Artificial Neural Network controller, ANN) and conventional controller technique (PID and LQR) for an inverted pendulum system
- ii. To make comparison between artificial intelligence controller technique and conventional controller technique
- iii. To design a graphic user interface (GUI) for the inverted pendulum system simulation

## 1.3 Scopes of Works

- i. Determine the mathematical model for an inverted pendulum system.
- ii. Design a controller using artificial intelligence technique (FLC and ANN) and conventional technique (PID and LQR)
- iii. Simulate the controllers using Matlab and conclude the best controller based on the simulation results
- iv. Design inverted pendulum system animation using Matlab

## 1.4 Research Methodology

1. Conducting literature review to understand the concept of an inverted pendulum system
2. Searching out previous and current projects of an inverted pendulum system, identifying problem faced by previous and current researcher and identifying suitable technique of designing the controllers
3. Defining mathematical model for an inverted pendulum system
4. Defining mathematical model of the controllers
5. Study the Matlab programming language, graphical user interface and simulink
6. Designing and writing Matlab program to simulate and animate the system
7. Analyze the process, Acquire control rules from experience operator and simulate the FLC, ANN, PID and LQR

All the methodology above can be summarized as shown Figure 1.1

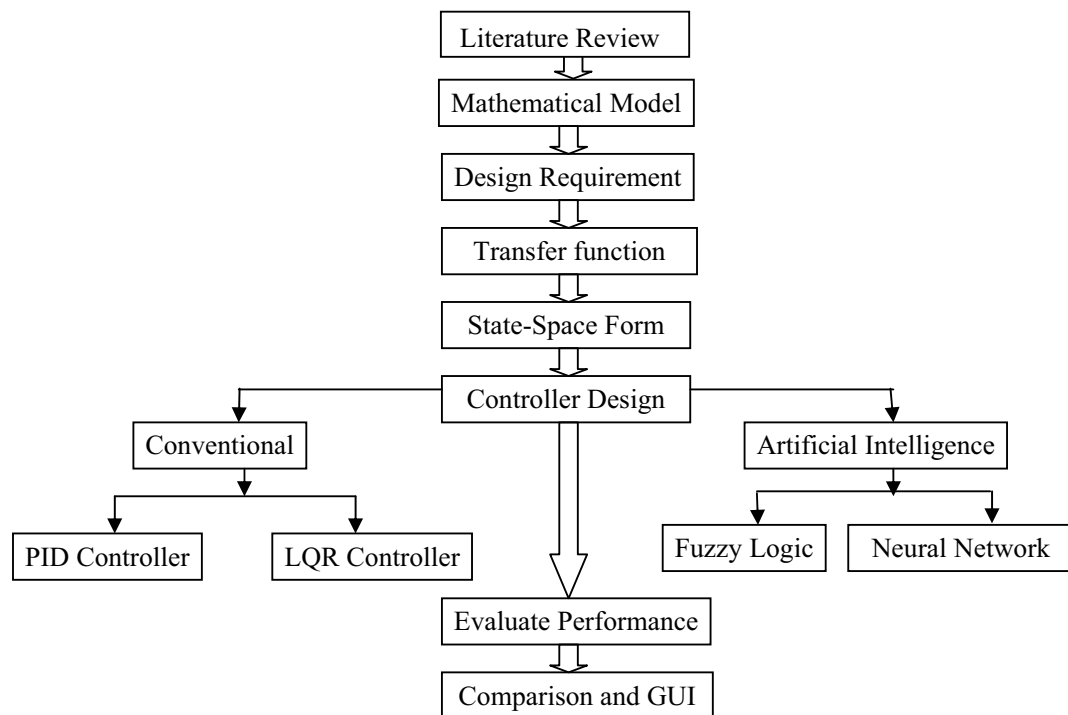


Figure 1.1: Flow chart of research methodology

## 1.5 Thesis Outline

This report consists of eight chapters including this chapter. The scope of each chapter is explained as stated below:

### Chapter 1

This chapter gives the introduction to the project report, objectives, scopes of works and methodology taken.

### Chapter 2

This chapter discusses modeling of an inverted pendulum. It is contained the derivation in mathematical modeling for the dynamic of the inverted pendulum system, including the nonlinear and linearized equations. It consists of development of an inverted pendulum model, followed by its transfer function and state-space representations. The modeling technique in Simulink is also discussed.

### Chapter 3

This chapter discusses the theory and application of PID controller of this project. Both transfer function and state-space models are used to analyze the controller to solve the inverted pendulum problem.

### Chapter 4

This chapter proposes LQR control method in controlling the inverted pendulum system by applying the state-space representation. There are also a few modifications that have been done to meet the design requirements and to improve the results.



## Chapter 5

This chapter proposes the Fuzzy Logic Controller (FLC) for the inverted pendulum system. It also describes some theoretical background of FLC such as the introduction and types of fuzzy. The proposed FLC characteristics for the inverted pendulum are also discussed.

## Chapter 6

This chapter discusses the Artificial Neural Network controller (ANN) of inverted pendulum system. It also includes the introduction, advantages of ANN's, types of learning, neural network structures, activation functions, learning algorithms, forward modeling of inverted pendulum using neural network, and learning rules

## Chapter 7

This chapter displays the results of the closed loop system of the inverted pendulum, the results of the PID and LQR controllers, and also the results of the Fuzzy Logic controller and Artificial Neural Network controller.

## Chapter 8

This last chapter presents the overall discussion and conclusion of this project. A few recommendations and suggestions also have been included for the future work.